Foundations of Intensional Semantics
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For my parents

לזכר אבי וบท פינפל המAsyncResult והיו
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We began working on this book in 2000 in order to address several issues in the foundations of intensional logic and computational semantics that we felt had been neglected for too long in the these fields. Both of us had been thinking along parallel lines within different formal frameworks, and we wanted to explore the possibility of combining our respective approaches into a unified theory. In the course of writing the book, our ideas evolved considerably under pressure of joint research and helpful critical responses from colleagues. What we had originally envisaged as a one- or two-year project turned into a four-and-a-half-year odyssey through some of the most complex and recalcitrant problems in type theory and intensional semantics of natural language. We discovered that these problems are far more difficult than we had originally appreciated, and we came to understand why much of the field preferred to set them aside in favour of more tractable and empirically accessible questions.

We cannot claim that this book provides definitive solutions to the foundational questions that we set out to deal with, and, in many ways, the proposals that we present in this monograph represent work in progress. However, we do think that the theory of intensional semantics that we develop in this study offers a promising framework for a computationally viable semantic
representation language, and we hope that this study will stimu-
late further work on the issues that we take up. Above all, we
have learned a great deal about the nature of the problems that
we address and the range of possible solutions to them. Our joint
work has significantly enriched our understanding of the role of
foundational issues in the development of an adequate semantic
theory.

Earlier versions of the ideas discussed in this book were pre-
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List of Abbreviations

ACE          antecedent contained ellipsis
BNF          Backus–Naur Form
CCC          Closed Cartesian Category
CG           Categorial Grammar
DRT          Discourse Representation Theory
FIL          Fine-grained Intensional Logic
GQ           Generalized Quantifier
HOU          higher-order unification
HPSG         Head-Driven Phrase Structure Grammar
IHOL         intensional higher-order logic
IL           intensional logic
ILF          interpreted logical form
ITT          intensional theory of types
LF           Logical Form
MLTT         Martin-Löf Type Theory
MS           Montague Semantics
NP           noun phrase
PLU          Logic Unplugged
PT           Property Theory
PTCT         Property Theory with Curry Typing
STT          Simple Theory of Types
UF           ultra filter
UT           Universal Type
UTIL         Untyped Intensional Logic
VP           verb phrase
The past 30 years have seen a vast expansion of research in formal and computational semantics of natural language. Much of this work has consisted in applications, extensions, and revisions of Montague’s (1974) model of intensional semantics. This model has provided a remarkably enduring and effective framework for developing analyses of complex properties of natural language interpretation. It remains one of the most rigorously specified, elegant theories of meaning yet formulated. However, it suffers from a number of formal and empirical limitations, and the recognition of these shortcomings has provoked fruitful investigations of alternative approaches to Montague Semantics (MS). In most cases, these alternatives have been developed to address a particular set of problems with MS. They generally focus only on those aspects of the theory that prevent it from accommodating the phenomena with which they are concerned, while bypassing a systematic re-evaluation of its general architecture and foundational assumptions.\(^1\)

Two important post-MS approaches that have played a prominent role in shaping research in semantic theory during the past

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1 See Dowty et al. (1981) for a classic introduction to MS. See Lappin (2000b) and Lappin (2003) for discussion of the three post-MS approaches to semantic theory sketched here.
two decades are dynamic semantics (Chierchia 1995; Groenendijk and Stokhof 1990, 1991; Heim 1982; Kamp 1981; Kamp and Reyle 1993) and situation semantics (Barwise and Etchemendy 1990; Barwise and Perry 1983; Cooper 1996; Seligman and Moss 1997). Dynamic semantics is devoted to modelling evolving relations of anaphora that emerge in discourse, between pronouns and certain types of definite descriptions, and their noun phrase antecedents. **MS** specifies the interpretations of sentences statically and in isolation. Therefore, it cannot handle the dynamic intersentential anaphora of discourse. Dynamic semantic theories propose representation languages that capture anaphoric dependencies exhibited in discourse sequences such as *A student arrived, He entered the class,* and in the donkey sentence *Every man who owns a donkey beats it,* where these dependencies are beyond the expressive resources of **MS**.

Situation semantics is concerned with the fact that **MS** represents the meaning of an expression in terms of its denotation in a possible world. **MS** uses an entire world, corresponding to a fully specified set of elementary facts, as a parameter for characterizing meaning. This creates difficulties for adequate analyses of reference, definite descriptions, and quantified NPs. Situation semanticists argue that these problems can be overcome if one substitutes situations (partial worlds) for worlds as the primary parameter relative to which the denotation of a term and the truth-value of a proposition are determined. Situations are fragments of worlds, and they provide a device for formally representing the contribution of contextual parameters to interpretation. So, for example, a definite description like *the book* does not have a unique denotation in most possible worlds, and certainly not in the actual world. However, it can be used to refer uniquely in an appropriately specified situation. Similarly, *Every student handed in a good paper* is false in the actual world, but true for an intended situation.

A third important trend in post-**MS** semantics is the move to underspecified semantic representations which leave the relative scope relations of quantified NPs and other scope-defining expressions (quantificational adverbs and adjectives, and negation) undefined in the interpretation of a sentence (Blackburn
and Bos 2005; Bos 1995; Copestake et al. 1997; Crouch and van Genabith 1999; Dalrymple et al. 1999; Reyle 1993). The sentence *Every student wrote three programs for two courses* allows six possible scope readings corresponding to the six possible distinct linear orderings of its three quantified NP arguments. MS adopts the principle of compositionality, which requires that the meaning of an expression $E$ is a function of the meanings of its constituents and the syntactic structure of $E$. MS takes the relation between syntax and semantics to be a homomorphism that maps all the elements of a syntactic category to the same semantic type. This entails that a sentence with six possible scope readings is ambiguous between six syntactic structures, each of which yields a representation with a distinct ordering of quantifiers. This approach generates spurious syntactic ambiguities that are not independently motivated by syntactic factors. If a single underspecified representation is assigned to the sentence, then one syntactic structure is sufficient, and unmotivated ambiguities are avoided. The underspecified representation can be resolved in different ways in order to generate the six scope readings.

While each of these approaches has contributed significant insights into semantic phenomena that MS is not able to capture adequately, none of them offers a systematic alternative to the basic assumptions underlying MS’s notion of intension and its relation to modality. Nor do any of these approaches examine and revise the type theory within which MS encodes its intensional logic. Instead they each construct a task-specific logic that is intended to solve their own particular range of problems.

In this monograph we depart from the strategy of local revision that has driven much of post-MS research, and instead focus on the major design choices of the MS framework. We pursue two main objectives. First, we are concerned to show that it is necessary to characterize intensions independently of modality and treat them as basic elements of one’s model theory in order to avoid the unattractive consequences of MS’s reduction of intensional identity to logical equivalence. These consequences continue to infect most post-MS theories. Second, we intend to demonstrate that by adopting a radically revised type theory,